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Chapter -5

ATMOSPHERIC ENERGY

5.1. THE THERMODYNAMIC ENERGY EQUATION

We now turn to the third fundamental conservation principle, the conservation of energy as applied to a moving fluid element. The first law of thermodynamics is usually derived by considering a system in thermodynamic equilibrium, that is, a system that is initially at rest and after exchanging heat with its surroundings and doing work on the surroundings is again at rest. For such a system the first law states that *the change in internal energy of the system is equal to the difference between the heat added to the system and the work done by the system.*
A Lagrangian control volume consisting of a specified mass of fluid may be regarded as a thermodynamic system. However, unless the fluid is at rest, it will not be in thermodynamic equilibrium. Nevertheless, the first law of thermodynamics still applies. To show that this is the case, we note that the total thermodynamic energy of the control volume is considered to consist of the sum of the internal energy (due to the kinetic energy of the individual molecules) and the kinetic energy due to the macroscopic motion of the fluid. The rate of change of this total thermodynamic energy is equal to the rate of diabatic heating plus the rate at which work is done on the fluid parcel by external forces.

The energy required to set the atmosphere in motion and to maintain the air currents inspite of the effect of viscosity is ultimately of solar origin. When radiation is absorbed at the earth's surface or in the atmosphere, it appears as internal energy. One of the basic problems of atmospheric science is to determine how this internal energy is converted into other atmospheric forms of energy such as potential, kinetic and latent and to formulate the equations.

The Energy Equation:

One important relationship can be obtained from the equation of motion:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv - 2\Omega w \cos\phi + F_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(.5.1)$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.2)$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u \cos\phi - g + F_z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.3)$$

Upon multiplying the three equations (5.1), (5.2) and (5.3) respectively by u,v and w and adding it follows:

$$u\frac{du}{dt} = -\frac{u}{\rho}\frac{\partial p}{\partial x} + fuv - 2u\Omega w\cos\phi + uF_x$$

$$v\frac{dv}{dt} = -\frac{v}{\rho}\frac{\partial p}{\partial y} - fuv + vF_y$$

$$w\frac{dw}{dt} = -\frac{w}{\rho}\frac{\partial p}{\partial z} + 2\Omega uw\cos\phi - gw + wF_z$$

These can be written after adding and taking $u^2+v^2+w^2= c^2$ as

$$\underbrace{\frac{d}{dt}\left(\frac{C^2}{2} + gz\right)}_{A} + \underbrace{\alpha\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right)}_{B} + \underbrace{(uF_x + vF_y + wF_z)}_{C} = 0 \;\ldots\ldots\ldots(5.4)$$

Term ‘A’: The first term is called the specific kinetic energy and the second term (gz) in the brackets is called the specific gravitational potential energy of the atmospheric parcel.
Term ‘B’ represents the work done on the unit mass by the pressure field as the parcel crosses the isobaric surfaces under non geostrophic conditions.
Term ‘C’ represents the energy dissipated due to the frictional forces. This term is negative because it converts kinetic energy into heat. The coriolis terms vanished here implies that it doesn’t influence on the heat or energy change.

Equation (5.4) deals with only mechanical energy since it contains only motion of the air parcel. In order to include the thermal energy also we must use the first law of thermodynamics as:

$$\frac{dh}{dt} = Cv\frac{dT}{dt} + p\frac{d\alpha}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.5)$$

Where dh represents an increment in specific heat added to the parcel in time ‘dt’. On adding the equations (5.4) and (5.5) we get

$$\frac{dh}{dt} = \frac{d}{dt}\left(\frac{C^2}{2} + gz\right) + \alpha\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + Cv\frac{dT}{dt} + p\frac{d\alpha}{dt} - (uF_x + vF_y + wF_z)\ldots(5.6)$$

We know $\frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}$ or $u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \frac{dp}{dt} - \frac{\partial p}{\partial t}$

On re arranging we get

$$\frac{dh}{dt} = \frac{d}{dt}\left(\frac{C^2}{2} + gz + CvT + p\alpha\right) - \alpha\frac{\partial p}{\partial t} - (uF_x + vF_y + wF_z)\;\ldots\ldots\ldots\ldots\ldots\ldots(5.7)$$

$$\because \frac{d}{dt}(p\alpha) = p\frac{d\alpha}{dt} + \alpha\frac{dp}{dt}$$

Equation (5.7) is called the atmospheric energy equation.

i) If the motion is adiabatic, $\frac{dh}{dt} = 0$

ii) If the motion is frictionless, $uF_x + vF_y + wF_z = 0$

iii) If steady state exists, $\frac{\partial p}{\partial t} = 0$

Under adiabatic frictionless steady state conditions, the equation (5.7) reduces to

$$\frac{d}{dt}\left(\frac{C^2}{2} + gz + CvT + p\alpha\right) = 0$$

On integration yields to $\left(\frac{C^2}{2} + gz + CvT + p\alpha\right)$ = constant ....(5.8)

Which means this is constant along a particular streamline. Thus the constant may vary from one stream line to the other.

The four terms on the left of the equation (5.8) represent various forms of energy and this equation (5.8) says that the sum of all these energies of a parcel on a surface is constant.

When only Kinetic energy is considered $(CvT = 0)$, then the equation (5.8) turns out to be the special case of Bernoulli's equation which represents the flow of mass on a streamline

$$\left(\frac{C^2}{2} + gz + p\alpha\right) = \text{constant} \;..................(5.9)$$

As the parcel rises either over a tall building or a mountain, the pressure decreases. As the pressure decreases in order to maintain constancy, velocity should increase at the peak. So the streamlines are crowded over the mountain peak while they are spacing at the foot hill as shown in Fig.5.1.Note the decrease of thickness (D) of streamlines over the peak than that at the foot hill. This is one of the reasons for getting more winds on the mountain tops than on the foot hills.

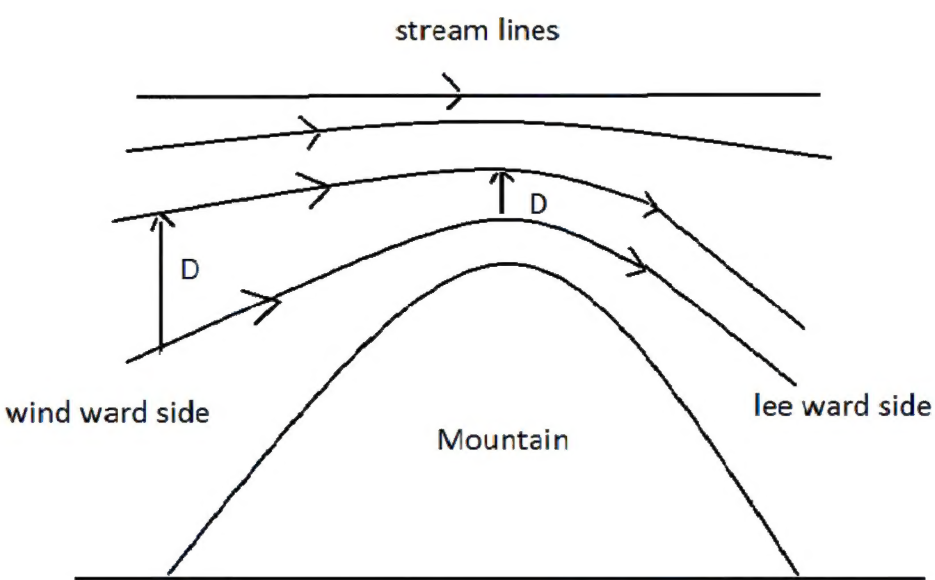

Fig.5.1. The passage of wind over the mountain. The thickness 'D' is small over peak than at the foot hill.